Bailey and Emma Use Direct Modeling to Solve an Addition Word Problem

This story is a part of the series:

***What's Next?* Stories of Teachers Engaging in Collaborative Inquiry Focused on Using Student Thinking to Inform Instructional Decisions**

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

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Introduction

This lesson focuses on students sharing their strategies to solve a word problem with multidigit numbers. It focuses on a progression of students sharing their strategies for direct modeling and counting.

Relevant Florida Mathematics Standards

MAFS.1.NBT.2.2. Understand that the two digits of a two-digit number represent amounts of tens and ones.

a. 10 can be thought of as a bundle of ten ones, called a "ten."
b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
d. Decompose two-digit numbers in multiple ways (e.g., 64 can be decomposed into 6 tens and 4 ones or into 5 tens and 14 ones).

MAFS.1.NBT.3.4. Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a "new" ten from some of the ones.

Background Information

This investigation was strongly informed by chapters two, three, six, and seven in *Children's Mathematics: Cognitively Guided Instruction.* These chapters provide background on addition and subtraction problem types as well as strategies that students use to solve these types of problems. In addition, chapter six provides some background information on base-ten number concepts. Chapter seven provides additional information on children's strategies when solving problems involving multidigit numbers.

Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. B. (2015). *Children's Mathematics: Cognitively Guided Instruction (2nd. ed.).* Portsmouth, NH: Heinemann.

Analyzing Student Thinking

In one-on-one interviews, a group of teachers posed the following four problems to each student in a class of first-grade students. Each teacher was assigned to one student in the class to interview individually with the goal of obtaining a good understanding of each child's mathematical understanding related to addition, subtraction, and place value on that day. The students had access to linking cubes, base ten blocks, pencils, and paper and were encouraged to use whatever tool or solution strategy they wanted to use. The interviewing teacher read each problem to the student, and the reading of the problem could be repeated as needed. After the student provided an answer, the student was asked to explain how he or she solved the problem.

Problem A
Pete had 20 rocks. Juan gave him 24 more rocks. How many rocks does Pete have now?

Problem B
Tylesha had 32 books. Her grandma gave her 25 more books. How many books does Tylesha have now?

Problem C
Mr. Jones had 40 cupcakes. He gave 20 cupcakes to the students in his class. How many cupcakes does Mr. Jones have now?

Problem D
Maria had 35 jellybeans. Her dad gave her 27 more jellybeans. How many jellybeans does Maria have now?

The following provides a quick summary of the working definitions of the student strategy categories that are discussed below.

The teachers discussed an observation that their textbooks encourage students to start operating on the digits in the ones place first, but the students in the interviews all started with the digits in the tens place before dealing with the digits in the ones place.

Multidigit Computation Strategies[1]

A student who uses a *direct modeling with ones* strategy represents each multidigit number in the problem as a set of ones using manipulatives or pictures to model the story in the problem and then counts the objects or pictures to determine the answer. In Problem B, for instance, a student using a *direct modeling with ones* strategy may create a set of 32 objects, create another set of 25 objects, put the two sets of objects together, and count each object in the full set by ones.

A student who uses a *counting by ones* strategy to solve this Problem B will do so without physically representing each quantity in the problem. Fingers, objects, or tally marks are often used to keep track of the number of counts. In Problem B, a student using a *counting by ones* strategy might say "32" and proceed to count forward by ones 25 numbers.

A student who uses a *directing modeling with tens* strategy represents each multidigit number in the problem using manipulatives or pictures that reflect the base-ten structure of the number system (e.g., with base-ten blocks or base-ten pictures). Then, the student counts the objects or pictures by tens and ones to determine the answer. In Problem B, the student could use base-ten blocks to model both 32 and 25 by representing 32 with three ten rods and two unit cubes then model 25 with two ten rods and five unit cubes. The student could then count the number of objects, either by ones or by tens and ones. For example, the student might say, "10, 20, 30, 40, 50, 51, 52, 53, 54, 55, 56, 57." (The student may also count the ten rods by ones and consider five ten rods to be 50.) If the student counts from 1 to 57 by ones, the student's strategy is still considered *directing modeling with tens*, because the student created a representation involving groups of tens and ones.

A student who uses a *combining the same units* strategy operates on the tens and ones separately, and the partial sums or differences are then combined to get a final result. In Problem B, the student might think about adding three tens and two tens to make 50, and adding two and five to make seven. The student might then say, "50 + 7 is 57".

The student who uses an *incrementing* strategy determines the answer by increasing or decreasing partial sums or differences. In Problem B, a student using an incrementing strategy might say, "30 + 20 is 50, 50 + 5 is 55, and 55 + 2 is 57."

After interviewing the students in the class and reflecting on the strategies the students used to solve the problems, the teachers sorted students' strategies for Problem B as shown in Figure 1.

1 The descriptions of strategies presented here are the current descriptions used by our team, and we consider them to be fluid, as our understanding of these ideas continues to evolve. For a more detailed discussion of these terms, consider reading Carpenter et al. (2015).

As you will notice in Figure 1, no students were placed in the category *counts on by ones*. The category was included, because the teachers anticipated—before giving the problem to students—that some students would start counting at 32 and count by ones (33, 34, 35, etc.) 25 counts to get to the answer of 57, but none of the students in the class actually used that strategy. Students were sorted into two different categories of *direct modeling*: those who modeled with only ones and those who modeled with tens and ones. The teachers mentioned in discussion that many of the students needed help with counting by ones (and also counting by tens) when they were modeling the problem. The discussion also included some students who started counting the tens first and then switched to counting the ones first. The teachers discussed an observation that their textbooks encourage students to start operating on the digits in the ones place first, but the students in the interviews all started with the digits in the tens place before dealing with the digits in the ones place.

Learning Goals

On the basis of how the students solved Problem B, the teachers determined they should use the following pedagogical goal for the new lesson:

> *Have selected students share the strategy they used to solve Problem B to provide an opportunity for students to learn additional strategies from their peers in an effort to help build fluency with two-digit addition problems.*

The teachers noticed that many students used a *direct modeling with ones* strategy, whereas many others used *direct modeling with tens*. They agreed that today might be a good time for students who were *direct modeling with ones* to hear from students who used a *direct modeling with tens* strategy. The teachers thought those students might benefit from hearing other students who counted by tens and ones. Finally, those students who struggled with the counting sequence when counting by ones could benefit from hearing and helping other students count aloud.

Planning for the Lesson

The teachers worked to develop the following word problem:

> *Jason had 34 pencils. His grandma gave him 23 more pencils. How many pencils does Jason have now?*

Rationale for the problem selected

The teachers chose to use a *join result unknown* problem. This is one of the easiest types of addition word problems, and the class had already demonstrated that they could successfully solve such problems. They decided to stick with this simple problem type so that students could focus on the numbers and the counting strategies rather than getting bogged down with more cognitively challenging situations in *compare, change unknown*, or *start unknown* situations. The num-

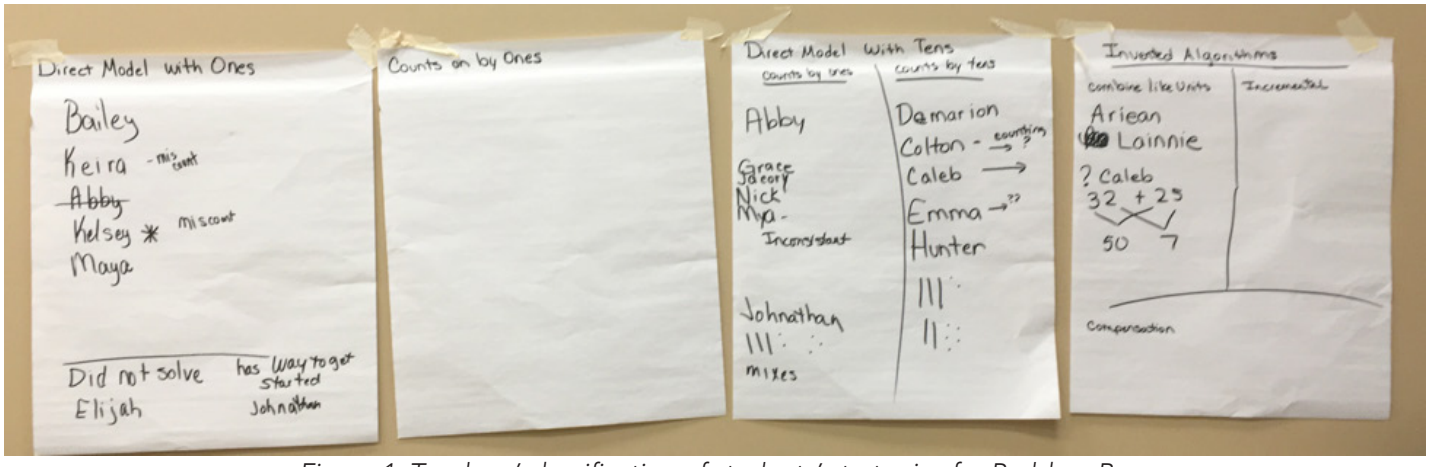

Figure 1. Teachers' classification of students' strategies for Problem B.

bers 34 and 23 were chosen in an effort to pose a problem that would lend itself to counting by tens and ones without having collections of more than ten ones to have to convert to a group of ten and some ones.

The teachers then analyzed the classification chart (Figure 1) and identified those students who would be ideal for sharing their strategy from Problem B in the interview with the class. They decided that, as students shared their strategies, an effort would be made to connect the ideas presented in the various strategies across the progression. Students who struggled to count would be asked to listen to others counting the objects in their models and would be asked to count verbally along with their peers.

On the basis of their analysis of the strategies students used in the previous problem, the teachers worked to develop a sequence for students to share their strategies for solving Problem B.

The teachers determined that the students would share in the following order:

1. Bailey—This student was selected to share first, because she used *direct modeling with ones* strategy, and the teachers thought Keira might benefit from hearing a student count by ones.

2. Emma would share next, because she used a *direct modeling with tens* strategy and also counted by tens. She would build her model and then ask one of the students who counted by ones to count Emma's model. Then, Emma would share how she counted her model. Abby would be invited to participate in the conversation, because she was unsure of how to build 32.

If time permitted, students who used *invented algorithms* would be asked to share their strategies.

As students share their strategies, the teacher will listen for opportunities to make connections between different students' strategies. In addition, the teacher will take note of those students who were making their own connections to ways their strategy is similar to or different from the shared strategies.

While students solve the new problem, the teacher will identify which students to ask to share their thinking with the rest of the class at the end of the lesson. The teacher will look specifically for variations on the theme of *direct modeling with tens*. The teacher will also make note of students who use a strategy different from that they previously used.

Lesson Plan

A reminder—on the basis of how the students solved Problem B, teachers determined they would use the following pedagogical goal to guide decisions during the classroom lesson:

> *Have selected students share their strategies they used to solve Problem B to provide an opportunity for students to learn additional strategies from their peers in an effort to help build fluency with two-digit addition problems.*

1. Call students to sit on the floor (or rug). Remind students of the behavior expected of them while they sit on the floor.

2. Say, "You solved some problems earlier, and you used several different strategies to solve them. We are going to look at one of the problems you solved, and some of you will get to share your thinking so that we learn from your ideas. Think about how you solved the following problem."

3. Display the problem for students to see. Then read the problem aloud to the class.

> *Tylesha had 32 books. Her grandma gave her 25 more books. How many books does Tylesha have now?*

4. Say, "Think about how you solved that problem. Can you read the problem with me?" Read the problem aloud again with students also reading verbally.

5. Ask the following questions while providing students sufficient time to discuss the answers and share their thinking.

a. How many books did Tylesha start with? [Check with Elijah and Johnathan, because they did not seem to make sense of the problem in interviews.]

b. How many books did grandma give to her? [Check with Elijah and Johnathan, because they did not seem to make sense of the problem in interviews.]

6. Write the numbers on the board or chart paper so that students can remember the numbers from the problem.

7. Ask students, "What do we need to find out in the problem?" [Once again, check with Elijah and Johnathan, because they did not seem to make sense of the problem in interviews.]

8. Say, "Now, listen carefully as students share their strategies. As you listen, think about how the strategies being shared are the same as or different from the strategy you used."

9. One at a time, ask Bailey and Emma to share their strategies. As students share, listen for opportunities for the students sitting on the rug to be able to come up and help the student articulate ideas. For example, if Bailey is having difficulty showing how she modeled the problem, ask another student who had used a *direct modeling with ones* strategy to help Bailey explain her model. Doing so will provide opportunities for students to learn from and help each other. Be careful to make sure Bailey remains involved in the discussion. When possible, ask questions related to how some strategies are similar and/or different.

10. Have students go quietly back to their seats.

11. Provide students with the following problem either displayed for the class or on a sheet of paper. Also, provide students with manipulatives and paper and pencil (or marker).

Jason had 34 pencils. His grandma gave him 25 more pencils. How many pencils does Jason have now?

12. Read the problem aloud to the class. Remind students that they may use whatever strategy makes the most sense to them.

13. Provide time for students to solve the problem.

14. While students work on solving the problem, circulate around the room and identify those students who would be good candidates for sharing their strategies on this new problem. Pay particular attention to those students you anticipate will move beyond the strategy used on Problem B. For example, pay attention to whether or not Grace and Abby count by ones or tens if they use a *direct modeling with tens* strategy again on this new problem. Also, listen to those students who struggled with counting by ones in Problem B. While circulating, stop and ask any clarifying questions in order to gain a better understanding of students' strategies and thinking. Make note of any students using the following strategies:

- Direct modeling with ones
- Counting by ones
- Direct modeling with tens
- Combining the same units
- Incrementing

Clarifying questions that could be asked include:

- What does this set of objects represent?
- How did you count the objects? Can you count out loud for me?
- I see that you wrote 30+20. What do those numbers represent?

15. Identify three to four students to share their strategy with the class in the closing portion of the lesson. Plan to sequence the order of their sharing from least sophisticated (e.g., *direct modeling with ones*) to more sophisticated (e.g., *direct modeling with tens*).

16. Call students back to the floor (rug) along with their student work (if students wrote their thinking

on paper). Remind students of behavior expectations.

17. Say, "You just solved a problem, and you used several different strategies to solve it. We are going to look at the problem you solved, and some of you will get to share your strategy so that we can hear more about how you were thinking of the problem. Think about how you solved the following problem."

18. Display the problem for students to see.

Jason had 34 pencils. His grandma gave him 25 more pencils. How many pencils does Jason have now?

19. Say or ask the following prompts while allowing students time to respond.

a. Think about how you solved that problem. Can you read the problem with me? Read the problem aloud with students.

b. How many pencils did Jason have to start with? Call on students who struggled to make sense of the problem.

c. How many pencils did his grandma give him? Call on students who struggled to make sense of the problem.

d. What did we need to find out in the problem? Call on students who struggled to make sense of the problem.

e. Now, listen carefully as students share their strategies. As you listen, think about how the strategies being shared are the same as or different from the strategy you used.

20. Call students up one at a time to share their strategy. As students share, listen for opportunities for the students sitting on the floor to be able to come up and help the sharing student if he or she is struggling. Doing so will provide opportunities for students to learn from and help each other.

Consider having students share strategies in the following order:

1. *Direct modeling with ones*
2. *Direct modeling with tens*
   a. *Counting by ones*
   b. *Counting by tens and ones*
3. *Combining the same units*
4. *Incrementing*

Similar questions and prompts can be used to help those students who are using a counting by ones strategy to move on to a *counting by tens* strategy.

Reflection

After the lesson, the teachers gathered to discuss and reflect. The following paragraphs outline some of the things they noticed from the lesson they observed.

The students in general made progress in their ability to share their own strategies and in relating their own strategies to those used by their peers. The classroom teacher noted that the students were not accustomed to sharing their strategies, and she thought that getting them to talk about their thinking was a struggle.

The teachers thought that the discussion and learning opportunities during the sharing portion of the lesson were beneficial. They referenced Keira as an example when she used a *direct modeling with ones* strategy but miscounted on Problem B. With the new problem, she then began to touch cubes as she counted, which may have been influenced by the experience of watching Bailey count her cubes.

Another example was seen in Problem B when Abby and Jacroy both counted by ones to determine their answer. On the new problem during the lesson, both of these children used a *directing modeling with tens* strategy. Both Johnathan and Elijah, who were unable to solve Problem B during the interviews, were able to successfully solve the new problem. Both of them used a *direct modeling with ones* strategy.

In discussion, the teachers agreed that this class is ready to solving problems involving numbers that will require students to make another ten, such as a *join result unknown problem* with the numbers 29 and 36. This problem will be more difficult than the problems the students solved today and will further challenge them to think about grouping tens and ones. In addition, the teachers conjectured that students might benefit from more opportunities to share their strategies with the class in the future. They seemed to learn from one another on this day, and their sharing provided good formative assessment opportunities for the teacher. Further opportunities (and expectations) for students to articulate their thinking and to be held accountable for listening to one another may increase both their expressive and their receptive communication abilities. In other words, being expected to share their thinking more often will give them practice explaining their thinking, listening to other ways of thinking, and making connections between different strategies and representations.

What's Next?

Stories of teachers engaging in collaborative inquiry focused on using student thinking to inform instructional decisions

What's Next? is a collection of stories documenting professional development experiences shared by elementary teachers working collaboratively to study the complex process of teaching and learning mathematics. Each story in the collection describes practicing teachers studying the thinking processes of real students and using what they learn about those students to make decisions and try to help advance those students' understanding on that day.

The teachers in each story start by learning about how individual students are solving a set of mathematics problems. They use this freshly gathered knowledge of student thinking to develop near-term learning goals for students and a lesson plan tailored to specific students on that specific day. One of the teachers implements the planned lesson while the other teachers observe in real time. The teachers then gather to discuss and reflect on their observations and insights.

In these lessons, the practice of teaching is slowed way down. The stories tell of teachers who are studying student thinking and using that information to plan and implement instructional decisions at a pace that is much slower than it occurs in daily practice. The stories in this collection also depict many aspects in common with formative assessment and lesson study, both of which are a process and not an outcome.

The stories depict real situations that occurred in real time and include both successes and shortcomings. We hope that the stories may be studied and discussed by interested educators so that the lessons and ideas experiences of these teachers and instructional coaches may contribute to additional learning and sharing among other interested teachers.

Learn more about these and other stories at **http://www.teachingisproblemsolving.org/**